Given name:____________________ **Family name:**________________________

Student number:____________________ **Signature:**________________________

UNIVERSITY OF TORONTO
Faculty of Arts and Science

STA 3431 (Monte Carlo Methods)
IN-CLASS TEST
November 20, 2017, 10:10 a.m.

Duration: 100 minutes. Total points: 40.

Aids allowed: NONE.

This examination paper consists of **5** single-sided pages (including this cover page), and **7** questions. The backs of the pages can be used to continue an answer (be sure to INDICATE THIS), or as scrap paper. The value of each question is indicated in [square-brackets].

Be sure to EXPLAIN all of your answers clearly!

1. For each of the following choices of linear congruential generator parameters, specify (with explanation) whether or not the generator has full period (i.e. period m).

   (a) [2] $m = 27$, $a = b = 5$.

   (b) [2] $m = 27$, $a = 4$, $b = 6$.

   (c) [2] $m = 32$, $a = 3$, $b = 5$.

2. [5] Suppose U is a random variable having the Uniform[0,1] probability distribution. Let $Y = -3\log(U)$ (where "log" is in base e). Compute, with proof, $\mathbf{P}(Y > y)$ for all $y \geq 0$, and identify the distribution of Y by name.

3. [5] Suppose $U_1, U_2, \ldots, U_M$ is a sequence of i.i.d. Uniform[0,1] random variables (for some large integer M). Explain (with words and equations) how to use the U_i to provide a good estimate of the two-dimensional integral $\int_0^1 \int_2^6 \sin(\sqrt{x + y^2})\, dy\, dx$. (You do not need to provide actual computer code, and you do not need specify standard errors.)

4. [6] Let $g(x) = e^{\sin x}\, \mathbf{1}_{4<x<7}$. Write (with explanation) an R program to run a rejection sampler to create a vector "`xlist`" of i.i.d. samples from the density proportional to g. [You may use R's "`runif(n, min, max)`" command to generate uniform random samples.]

5. [3] Let $\{X_n\}$ be a run of an MCMC algorithm with stationary distribution π, and let h be a real-valued functional. Suppose that the stationary lag-k correlations of $\{h(X_n)\}$ are given by $\rho_k := \mathrm{Corr}_\pi(h(X_0), h(X_k)) = 4^{-k}$. Compute the value of "varfact" (i.e., the integrated auto-correlation time) when estimating h with this algorithm.

6. Consider an MCMC algorithm, with target density π. Define with a formula, and also describe in words, each of the following:

   (a) [3] The distance function $D(x, n)$.

   (b) [3] The algorithm being "ergodic".

   (c) [3] The algorithm being "geometrically ergodic".

7. [6] Let $\{X_n\}$ be a run of an independence sampler on $\mathbf{R}$ with target density $\pi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$, and proposal density $q(x) = \frac{1}{2} e^{-|x|}$. Find (with explanation) a value $\rho < 1$ such that if the algorithm is started with $X_0 = 5$, then for all $n \in \mathbf{N}$,

$$\left| \mathbf{P}(X_n < 0) - \frac{1}{2} \right| \leq \rho^n .$$

End of examination
Total pages: 5
Total points: 40