STA130, Winter 2017: Midterm

(100 minutes; 6 questions; 8 pages; total points = 66)

| LAST NAME: | GIVEN NAMES: | STUDENT #: |
|------------|--------------|------------|
| | | |

Tutorial (circle one):

| BL113-Jessica | BL114-Anthony | RW141-Fang |
|---------------|-----------------|--------------|
| UC248-Kuan | UC257-Courtneay | SS2114-Saeha |

Do not open this booklet until told to do so. Answer all questions. Aids allowed: a simple non-programmable calculator.

Point values for each question are indicated [in square brackets]. You may continue on the back of the page if needed (write "OVER"). A standard normal probability table is included at the end.

DO NOT WRITE BELOW THIS LINE.

| Question | Score |
|--------------|-------|
| 1 | /4 |
| 2 (a) | /3 |
| 2(b) | /3 |
| 2(c) | /3 |
| 3(a) | /4 |
| 3(b) | /4 |
| 3(c) | /4 |
| 4(a) | /3 |
| 4(b) | /3 |
| 4(c) | /3 |
| 4(d) | /3 |
| 4(e) | /3 |
| 4(f) | /2 |

| Question | Score |
|-------------|-------|
| 5(a) | /1 |
| 5(b) | /1 |
| 5(c) | /3 |
| 5(d) | /4 |
| 6(a) | /2 |
| 6(b) | /2 |
| 6(c) | /2 |
| 6(d) | /4 |
| 6(e) | /2 |
| 6(f) | /3 |
| TOTAL: | /66 |

1. [4] In three or four complete English sentences, explain what a P-value is, and what it is used for.

2. Suppose Y is a random quantity having normal probabilities, with mean 20 and variance 16.

(a) [3] Compute P(Y < 26). [Hint: don't forget the standard normal probability table included at the end of this test.]

(b) [3] Compute P(Y < 18).

(c) [3] Provide an estimate of P(Y > 100). [Hint: if a value is too large for the standard normal table, then what can you conclude?]

3. Suppose 50 people each have their own deck of cards. Each person picks either the 2 or 3 or 5 or 6 of Spades, with probability 1/4 each. Let X_1 be the card chosen by the <u>first</u> person. Let T be the <u>sum</u> of all 50 cards, and let Y = T/50 be the <u>average</u> of all 50 cards.

(a) [4] For X_1 , compute the expected value $E(X_1)$ and the variance $Var(X_1)$.



(b) [4] For T, compute the expected value E(T) and the variance Var(T).

(c) [4] For Y, compute the expected value E(Y) and the variance Var(Y).

4. The champion New England Patriots football team won 14 of the 16 games they played during this year's regular NFL season. Suppose we wish to test the null hypothesis that their games were all just random luck with probability 1/2 of winning each game, versus the alternative hypothesis that they had probability <u>more</u> than 1/2 of winning each game.

(a) [3] Under this null hypothesis, what would be the mean and variance and sd for the number of games a team would win (out of 16 games total)?

(b) [3] Under this null hypothesis, what would be the probability of a team winning all 16 games (out of 16 games total)?

(c) [3] Under this null hypothesis, what would be the probability of a team winning exactly 15 games (out of 16 games total)? [Hint: How many sequences are there corresponding to 15 W and 1 L?]

(d) [3] Under this null hypothesis, what would be the probability of a team winning exactly 14 games (out of 16 games total)? [Hint: You may use the fact that $\binom{16}{14} = 120$.]

(e) [3] Using all of the above information, what is the P-value for this hypothesis test?

(f) [2] What can we conclude from this P-value? (State your conclusion clearly, using complete English sentences.)

5. A recent study¹ was reported with such headlines² as "Patients treated by female doctors less likely to die, study shows". The study examined 415,559 elderly patients who saw female doctors, of which 10.82% of them died within 30 days. It also examined 1,200,296 elderly patients who saw male doctors, of which 11.49% of them died within 30 days.

(a) [1] Based on the above, what (approximately) is the actual <u>number</u> of patients who saw <u>female</u> doctors and then died within 30 days?

(b) [1] Based on the above, what (approximately) is the actual <u>number</u> of patients who saw <u>male</u> doctors and then died within 30 days?

(c) [3] Under the null hypothesis that 10.9% of all patients who see <u>female</u> doctors will die within 30 days, compute the mean and variance and standard deviation for the number of patients out of 415, 559 who would die within 30 days of seeing a female doctor.

(d) [4] Compute (with explanation) a P-value for the null hypothesis that 10.9% of all patients who see <u>female</u> doctors will die within 30 days, versus the alternative hypothesis that it's <u>less</u> than 10.9%. What can we conclude from this?

 $^{^{1}} http://jamanetwork.com/journals/jamainternalmedicine/article-abstract/2593255$

²http://www.medicalnewstoday.com/articles/314912.php

6. Consider again the study described in Question 5.

(a) [2] Write down the general formula, in terms of the unknown true fractions p_1 and p_2 , for the sd of the <u>difference</u> $\hat{p}_2 - \hat{p}_1$ between the observed fraction of deaths within 30 days after seeing a female doctor, minus the observed fraction after seeing a male doctor.

(b) [2] Estimate the sd in part (a) using the <u>bold</u> option.

(c) [2] Estimate the sd in part (a) using the <u>conservative</u> option. How is it different?

(d) [4] Using the bold option, compute (with explanation) an estimate of the P-value for the null hypothesis that the <u>same</u> fraction of patients will die within 30 days whether they see a female or a male doctor, versus the alternative hypothesis that the fractions are different (either larger <u>or</u> smaller, i.e. two-sided).

(e) [2] State your final conclusion from part (d) in a clear, complete English sentence.

(f) [3] Has this study convinced you that seeing a female doctor makes people live longer? Why or why not? (There is no completely right or wrong answer here; just explain your <u>opinion</u> with some <u>reasons</u>.)



Standard Normal Probability Table

| | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |

[END OF EXAMINATION: total points = 66]