Name: ______________________________ Student Number: ____________________

STA2111F, Fall 2011, Mid-Term Test

(October 24, 2011. Duration: 60 minutes. Questions: 4. Pages: 3. Total points: 36.)

Note: You should fully explain your answers. Continue on the back if necessary.

1. Let $\Omega = \{1, 2, 3\}$. Let $\mathcal{F} = \Big\{\emptyset, \{1\}, \{2, 3\}, \{1, 2, 3\}\Big\}$, which you may assume is a σ-algebra. Let $\mathbf{P} : \mathcal{F} \to [0, 1]$ by $\mathbf{P}(\emptyset) = 0$, $\mathbf{P}(\{1\}) = 4/5$, $\mathbf{P}(\{2, 3\}) = 1/5$, and $\mathbf{P}(\{1, 2, 3\}) = 1$. Let $X, Y : \Omega \to \mathbf{R}$ by $X(1) = 5$, $X(2) = 10$, $X(3) = 10$, $Y(1) = 2$, $Y(2) = 4$, $Y(3) = 6$.

(a) [3 points] Verify that $\mathbf{P}$ is countably additive on $\mathcal{F}$.

(b) [3 points] Is X a valid random variable on $(\Omega, \mathcal{F}, \mathbf{P})$?

(c) [3 points] Is Y a valid random variable on $(\Omega, \mathcal{F}, \mathbf{P})$?

(d) [1 point] Compute $\mathbf{P}(X > 8)$.

2. [5 points] Let $(\Omega, \mathcal{F}, \mathbf{P})$ be as in Question 1. Let $\mathcal{G}$ be the collection of <u>all</u> subsets of Ω (so, $\mathcal{F} \subseteq \mathcal{G}$). Determine (with explanation) which <u>one</u> of the following statements is true (recalling that "extension from $\mathcal{F}$ to $\mathcal{G}$" means a countably additive probability measure on $\mathcal{G}$, which agrees with the original $\mathbf{P}$ when restricted to $\mathcal{F}$):

(i) $\mathbf{P}$ has no possible extension from $\mathcal{F}$ to $\mathcal{G}$,
or (ii) $\mathbf{P}$ has one unique extension from $\mathcal{F}$ to $\mathcal{G}$,
or (iii) $\mathbf{P}$ has more than one possible extension from $\mathcal{F}$ to $\mathcal{G}$.

3. [5 points] Let $(\Omega, \mathcal{F}, \mathbf{P})$ be any valid probability triple for which $\Omega = \{1, 2, 3, \ldots\}$, and $\mathcal{F}$ is the collection of all subsets of Ω. For each $n \in \mathbf{N}$, let $A_n = \{n, n+1, n+2, \ldots\}$. Is it necessarily true that $\lim_{n\to\infty} \mathbf{P}(A_n) = 0$? Why or why not?

4. Let $(\Omega, \mathcal{F}, \mathbf{P})$ be the probability triple defined by $\Omega = \{1, 2, 3, 4\}$, and $\mathcal{F}$ is the collection of all subsets of Ω, and $\mathbf{P}(\{1\}) = \mathbf{P}(\{2\}) = \mathbf{P}(\{3\}) = \mathbf{P}(\{4\}) = 1/4$. Let $A_n = \{1\}$ for n odd, and $A_n = \{2, 3\}$ for n even.

(a) [4 points] Are $A_1, A_2, A_3, \ldots$ independent?

(b) [4 points] Compute $\mathbf{P}\left(\liminf_{n\to\infty} A_n\right)$.

(c) [2 points] Compute $\liminf_{n\to\infty} \mathbf{P}(A_n)$.

(d) [2 points] Compute $\limsup_{n\to\infty} \mathbf{P}(A_n)$.

(e) [4 points] Compute $\mathbf{P}\left(\limsup_{n\to\infty} A_n\right)$.

[END]