

Probability, Statistics, and Murder

Jeffrey S. Rosenthal

Professor of Statistics
University of Toronto

jeff@math.toronto.edu

www.probability.ca

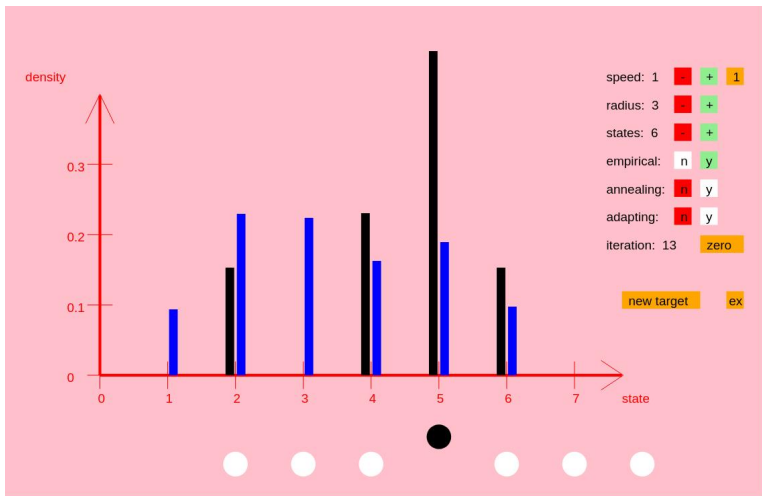
[@ProbabilityProf](#)

Presidential Invited Address

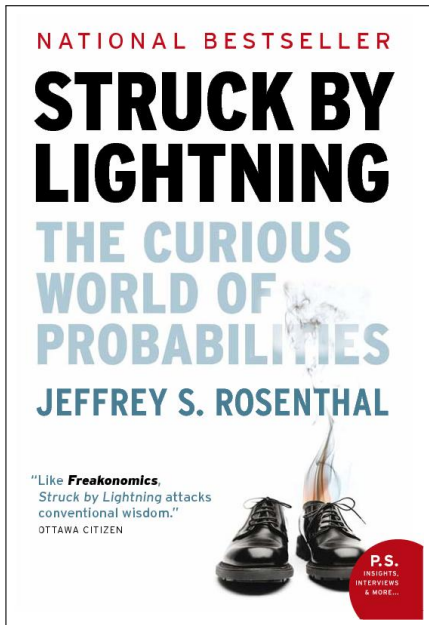
(SSC 2021 Annual Meeting, June 7, 2021)

What to Talk About?

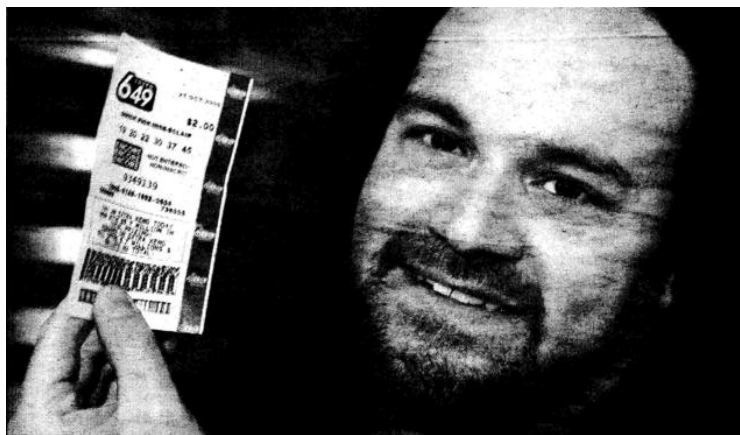
My main research area is theoretical analysis of MCMC algorithms.
But you've all heard me discuss that before ...



Instead: Statistics and the Law! It started with a book ...



Which Led to the Lottery Retailer Scandal



University of Toronto statistician Prof. Jeffrey Rosenthal holds up a 6/49 ticket. He says so many retailers have won prizes it raises suspicions.

Storekeepers lucky?

Lottery officials dismiss statistician's claim that retailers' wins defy odds

See IRAQ on page A16

welcoming country. A16

■ Editorial. A26

214 lottery 'insiders' won big bucks

Odds of Ontario results astronomical, investigation by CBC program reports

BY SHANNON KARI

More than two-hundred lottery "insiders" have won prizes of \$50,000 or more in Ontario since 1999, and more than two-thirds of these wins may have involved the deception of a customer who bought the ticket.

The allegation is made by the

CBC program *the fifth estate*, after an investigation into the number of "insider wins" in the province in the past seven years.

A statistical analysis of the number of insider wins concluded that fewer than 60 insiders, such as ticket retailers or clerks, should have won major prizes during the period

that was investigated.

The odds that the 214 insiders who claimed major prizes — \$50,000 or more — since 1999 won as a result of pure luck, is one in a trillion, trillion, trillion, trillion, said University of Toronto professor Jeffrey Rosenthal, who conducted the analysis.

The program airing tonight suggests this may be a problem across Canada and the United States.

See LOTTERY on page A10

Province to probe the win

Opposition wants outside investigator Statisticians lay bets there was fraud

**ROB FERGUSON
AND CURTIS RUSH**
STAFF REPORTERS

Queen's Park has ordered a probe of the Ontario Lottery and Gaming Corporation's security measures in the wake of allegations that an astonishing number of inside retailers have won prizes of \$50,000 or more since 1999.

"If there is anything we can do to strengthen that we certainly will act," David Caplan, the minister responsible for the corporation, said yesterday.

Caplan said he has ordered

frey Rosenthal's statistical analysis carried out for *the fifth estate*.

Rosenthal's numbers suggest that the real odds of 214 insiders claiming major prizes in those years are one in a trillion, trillion, trillion — that's a 1 followed by 48 zeroes.

In its Tuesday night report, CBC claimed that two-thirds of

wins by so-called insiders may have involved deception by unscrupulous vendors who stole customers' winning tickets.

Rosenthal, a statistician with the University of Toronto, claims the chances of so many vendors winning by chance are so small that "almost anything you can think of is more likely."

The odds that you'll be hit by lightning in the next year are one in 5 million; that you'll be killed in your next vehicle trip one in 7 million, according to

Another lottery gamble

Imagine that you are trusting enough to give your provincial lottery ticket to a store clerk to learn whether you have won. Imagine that, against astronomical odds, the ticket is worth a lot. The vast majority of retailers and clerks will tell you the truth. But, on the evidence of an episode of CBC's *the fifth estate* scheduled to air last night, a number will lie. They will say you won only a tiny prize and will collect the major one themselves. Yes, there are all sorts of mechanisms to prevent that from happening, including self-serve ticket checkers and machines that play different tunes depending on how much money a ticket has won, but it happens.

One case in Ontario has become public, though the Ontario Lottery and Gaming Corporation (OLGC) has worked hard to keep some details secret. Bob Edmonds, now 82, bought a ticket in 2001 worth \$250,000, but a variety-store owner told him he had won only a free ticket. Mr. Edmonds grew suspicious when he later read that the store owner had "won" the big prize. An OLGC investigator had concerns as well, but the corporation paid the retail-

tion incurred \$425,000 in legal costs, far exceeding the amount Mr. Edmonds should have won in 2001. And it required that the settlement be covered by a confidentiality clause — a demand that Mr. Edmonds's lawyer plans to contest in court next Monday, on the excellent grounds that an agency of government should not keep such secrets from the public.

The OLGC says the Edmonds case was "isolated" and insists it keeps a tight rein on those who sell its tickets (lottery mechanisms vary across Canada). The math would suggest otherwise. Initially, using an OLGC figure of 60,000 retailers and clerks in Ontario, *the fifth estate* asked Jeffrey Rosenthal, author of *Struck by Lightning: The Curious World of Probabilities*, what the odds were of those clerks winning nearly 200 times in the past seven years with an average prize of \$500,000. Dr. Rosenthal's answer: "one chance in a trillion, trillion, trillion, trillion." According to the CBC, the OLGC last week said it really had closer to 140,000 retailers and clerks; even at that, Dr. Rosenthal said, the chance of so many winning so much was

U
W
E
L
t
h

i
n
P
r
c
l
"V
i
s
a
f
c
i
o
g
e
p
e
a
f
a
r
c
o
t
h

T
r
m
T
t
i
c
p
o
c
T
A
A

t
o
c
a
m
o
t

Which Had Serious Consequences

Ontario Lottery chief fired

Mar. 24, 2007, 6:12 a.m.


5 comments



Ontario Lottery and Gaming Corporation: Ontario Lottery chief fired

 E-mail to a friend

 Printer-friendly

 Link to this story

Rating: ★★★★★

anonymity.

The head of the Ontario Lottery and Gaming Corporation was dismissed from the scandal-plagued organization on Friday, according to CBC News.

CEO Duncan Brown was escorted out of the lottery corporation's offices in Toronto, two sources told the CBC, speaking on condition of

Including Ontario Legislature Debate

Mr. Robert W. Runciman (Leeds-Grenville): My question is for the Minister of Public Infrastructure Renewal. Minister, today there are disturbing reports that over the last several years more than 200 lottery insiders have won prizes in excess of \$50,000. Jeffrey Rosenthal, a U of T professor, says that it's "extremely unlikely" these insiders would hit the jackpot that many times. The story, which is going to air on The Fifth Estate tonight, suggests that two thirds of these insider wins may have involved deception. Minister, can you tell us when you became aware of this issue and whether or not you plan to investigate the matter to ensure that Ontarians are not being defrauded of their rightful winnings?

Hon. David Caplan (Minister of Public Infrastructure Renewal, Deputy Government House Leader): There has been an allegation made, and I want all members to know that I take that very, very seriously. Ontario Lottery and Gaming is certainly committed to operating a business in a responsible and ethical manner and has some of the most stringent inside-win provisions of any organization

[And Ultimately to Criminal Charges](#)

\$12.5M lottery prize theft leads to 3 arrests

Last Updated: Wednesday, September 29, 2010 | 10:25 PM ET

Comments 462 Recommend 322

CBC News



The case of Kathleen Chung, who allegedly cashed the winning ticket at her brother's convenience store in Burlington in early 2004, was profiled by the CBC's Fifth Estate, triggering a report by the Ontario ombudsman. (CBC)

Three family members in the Toronto area have been charged in the theft of a \$12.5-million lottery prize, while police seek the rightful owner of the Lotto Super 7 ticket bought in 2003.

Two of the accused are a father and son who worked at a Burlington lottery outlet and were actively stealing tickets from customers, Ontario Provincial Police Commissioner Chris Lewis

Could the True Winners be Found?

\$12.5M lottery ticket stolen; police seeking rightful winner

Article

Photos (5)

Published On Thu Sep 30 2010

Email | Print |

Share



Curtis Rush and Jayme Poisson

51

Staff Reporters



Found them!

Lottery fraud victims claim \$12.5M prize – plus interest

Seven men with wraparound smiles claimed their seven-year-old \$12.5 million lottery win Thursday, which stands now at \$14.85 million.

Text size:   Reset



 Report an Error

 save to mystar



TANNIS TOOHEY / TORONTO STAR

[And It All Followed From Statistics](#)

Solving equations and solving crime

Toronto math prof proves his point when it comes to insiders winning lottery prizes

By **THANE BURNETT**, NATIONAL BUREAU

Last Updated: 21st February 2009, 2:36am

Story

Comments

Email Story | Print |

Size **A A A** | Report Typo

Share       +

For a guy who's never bought a lottery ticket at a corner store, Jeff Rosenthal has sure hit it big.

And lottery corporations across Canada would likely wish he would just stop playing their numbers so well.

Since 2006, the Toronto math professor has been the brain behind uncovering a suspected massive theft of lottery winnings by sticky fingered store clerks.

- Later cases in many other provinces, some U.S. states.
- Detailed article: www.probability.ca/lotteryscandal

[This Story Connected Me to the Legal World](#)

- Spoke at Toronto Police Services fraud conference, 2007. And:

Royal Canadian Mounted Police

Gazette magazine

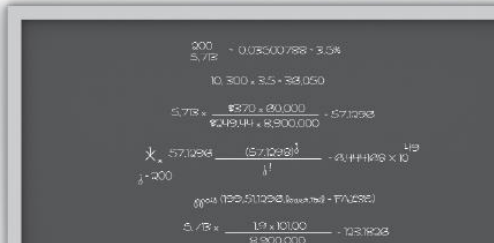
LOTTERY FRAUD

Solving crime using math

By Professor Jeffrey S. Rosenthal
Department of Statistics
University of Toronto

On the CBS television series NUMB3RS, crime-fighting mathematician Charlie Eppes boldly declares, "Everything is numbers!"

Well, that might be an exaggeration. But my involvement in a recent investigation into lottery fraud has convinced me that statistical analysis can indeed be used to uncover fraudulent behaviour that might otherwise pass undetected.



And Led to Some Academic Legal Research

- Law prof Albert Yoon: Which U.S. Supreme Court justices rely most on their law clerks to help them write their decisions?
- Idea: Using clerks would lead to more variable writing style.
- So: Measure the variability in the frequency of different “function words” (e.g. *all, have, not, than, with*).
- (cf. Mosteller and Wallace, JASA 1963, re Federalist Papers)
- Wrote programs, analysed the texts, produced rankings.
- Confirmed some previous beliefs. Found some new results too.
- Submitted to *JASA – Applications and Case Studies* . . .
- Published instead in *Annals of Applied Statistics*. (Quadfecta!)
- Companion piece in *Cornell Law Review*. (Told law friend . . .)
- Canadian Supreme Court: *University of Toronto Law Journal*
- And, this work must be important, because . . .

Justices Are Long on Words but Short on Guidance

A forthcoming [study](#) from two professors at the University of Toronto tried to identify the amount of ghostwriting on the court by developing software to analyze how justices' writing styles varied from opinion to opinion and term to term.

"A justice who wrote her own opinions would presumptively possess a less variable writing style than a justice who relied heavily on her law clerks," wrote Jeffrey S. Rosenthal and Albert H. Yoon, the authors of the study.

The opinions of Chief Justice Roberts and Justices Scalia and Breyer were less variable in this sense, and those of Justices Thomas, Ginsburg and Kennedy more so. The highest level of variability among justices who served since 1941 was in the opinions of Justice Sandra Day O'Connor, who retired in 2006.

Two Seventh Circuit judges known to write their own opinions, Judge Posner and Chief Judge Frank H. Easterbrook, have variability rates much lower than those of any current member of the Supreme Court.

Statistical Standards in Legal Cases

I gave talks to lawyers and judges: Canadian Institute for the Administration of Justice, Irish Supreme Court justices, . . .

Standard for criminal conviction: “Beyond a reasonable doubt.”

e.g. Ireland Public Prosecutions: “The judge or jury has to be convinced beyond a reasonable doubt that a person is guilty. It is not enough for them to think that the accused is probably guilty.”

Somewhere between “probably” and “certainty”. Huh? Between “< 100%” and “100%”? Human judgement! Expert testimony?

The New York Times

April 20, 2019

“An expert can say whatever they want,” said Simon Cole, the director of the registry and a professor of Criminology, Law and Society at [UC -Irvine](#).

That includes offering up invented odds like “one in a million” or “1 in 129,600,” the registry says.

Example: The case of Sally Clark

- Solicitor in Cheshire, England.
- Had two sons; each died in infancy.
- “cot death” (SIDS)? Or murder!?!
 - Testimony by paediatrician Sir Roy Meadow: “the odds against two cot deaths in the same family are 73 million to one”.
 - Convicted! Jailed! Vilified! Third son temporarily taken away!



Was “73 million to one” computed correctly?
And, was it the right thing to compute? No!

How did Meadow compute that “73 million to one”?

He said the probability of one child dying of SIDS was one in 8,543. Then for two children dying, he multiplied:
 $(1/8,543) \times (1/8,543) = 1/72,982,849 \approx 1/73,000,000.$

Clark Case: Valid Probability Calculation?

- Was the figure $1/8,543$ valid? Not really!

The overall probability of SIDS in the U.K. was (then) $1/1,303$.

Meadow “adjusted” for family circumstances that lower it (no smokers, parent employed, mother over 26), but ignored other factors which raise it (e.g. twice as likely for boys as girls).

- Multiplication: independent? No! SIDS tends to run in families, so a second SIDS case is about 7 times more likely.
- True probability $\approx 1 / 240,000$. Sufficient to convict?
- No! Multiple testing problem! Millions of families in the U.K. / World! Use a Bonferroni correction? (“Out of how many?”)
- Objections from Royal Statistical Society, Medical Council.
- Sally Clark was eventually acquitted, on second appeal, after more than three years in jail. But she never recovered psychologically, and died of alcohol poisoning four years later.
- Several other people’s convictions also overturned on appeal.

A Related Case: Waneta & Tim Hoyt (New York)

- Had five babies in 1965 – 1971. All died.
Ages (months): 3, 28, 1.5, 2.5, 2.5.

Pediatrician Alfred Steinschneider investigated, wrote 1972 article for journal *Pediatrics*.
Conclusion: “genetically-linked SIDS”.



- In 1977, they were allowed to adopt a son, who survived to adulthood.
- In 1985, some prosecutors and pathologists got suspicious, and investigated. Eventually, Waneta Hoyt confessed to suffocating all five children, to stop them from crying.
- She later “recanted” her confession, but was still convicted.
- Using the above factors, the probability should be about

$$1 / [(1303)^5 / 7^4] \doteq 1 / 1.5 \text{ trillion.}$$

Should this have led to an earlier conviction? Suspicions at least!

- Statistical evidence can indicate guilt . . . if you're careful.

A Case I was Involved With: Leighton Hay

- Accused as an accomplice in a 2002 murder.
- Witnesses: Hair was “two inch picky dreads”.
- But Hay was shaved nearly bald when arrested.
- Prosecution: He shaved his head afterwards!
- Evidence: Tiny hair clippings in a garbage bin and on an electric shaver in his home.
- Convicted in 2004. Fresh SCC appeal in 2011.
- Question: Were those clippings from a scalp?



Sample: Scalp hairs usually \leq 125 microns thick, but beard hairs often thicker. (They tried to exclude the 125 micron ones, too.)

My expert report: Of the 368 clippings collected, the number from a scalp was between 0 and 106 (29%), with the rest from a beard.

- Gave deposition. Cross-examined: aggressive! 2013 SCC 61 judgment: New trial! Hay released from jail. Not re-tried.

Another Case I was Involved With: Yuk Yuen Lee

- Accused in 2013 of running a marijuana grow-up in Toronto.
- Police seized 1378 + 2240 plants, all claimed to be marijuana.
- However, they only actually tested 2 + 1 of them (!).
- Convicted at trial. But what about the sentence?
- If more than 500 plants, then mandatory three-year jail term.

My expert report: The testing only showed that at least 138 + 16 of the plants were marijuana (exact 99% confidence interval).

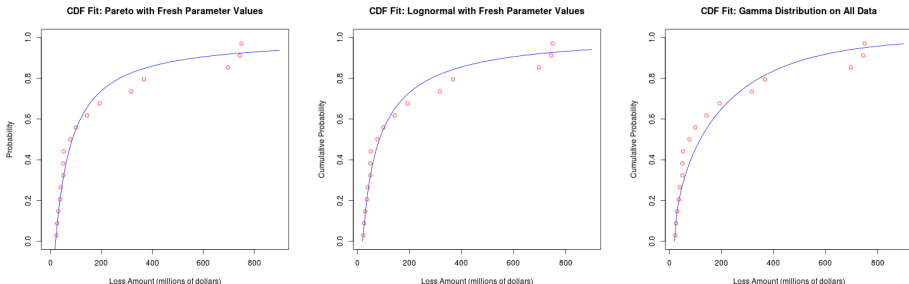
- Testimony. Cross-examination: attacks! (references ...)

2017 ONSC 2403 judgment: "Crown counsel suggests that Professor Rosenthal misrepresented the evidence [which] substantially undermined his credibility. ... I did not find Professor Rosenthal lacking in credibility. ... His evidence did not misrepresent in any way. ... I do not accept that the Crown has established the number of marijuana plants"

Sentence: Just the time already served. (Another case, too.)

A Commercial Case: Oil Shipping Liability

- Rail fees for transporting dangerous goods are based on the amount of “risk” they entail. How to estimate this?
- Measure used: “Total Exposure ($N\%$)”, i.e. the N^{th} percentile of damage values. Here $N = 99.7$ or 99.9 or 99.97 ; extreme events.
- But only 17 observations. So, they extrapolated. But different distributions/percentiles give very different estimates (factor > 30):



- Sent my memo. Received. Payment? Feedback? (Months later.)

Statistics and the Law: Reflections

- Lots of potential for academic statistical analysis of law data.
- In legal trials, statistics can be misused, to wrongly imply guilt.
- But they can also help to analyse and clarify evidence – good.
- Fact: I'm currently engaged in another case (delayed by covid).
Opposing “expert”: attacks! Statisticians are nicer! (Donation . . .)
- Be cautious before agreeing. (Pressure?) (Unpaid colleague!)
- Being a statistical expert witness can be: Interesting! Different!
Impactful! Satisfying! And . . . very well paid! (hourly or fixed?)
- But it can also be: Time-consuming! Annoying! Frustrating!
And, the “adversarial” system can be unpleasant – even nasty!
- And yet . . . if we statisticians don't do it, then who will?

Article www.probability.ca/justice, book Knock On Wood (ch. 19).

www.probability.ca / [@ProbabilityProf](https://twitter.com/ProbabilityProf) / jeff@math.toronto.edu